IPPP/05/19, DCPT/05/38
FERMILAB-PUB-05-195-T

Unconventional cosmology on the (thick) brane

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Abstract

We consider the cosmology of a thick codimension 1 brane. We obtain the matching conditions leading to the cosmological evolution equations and show that when one includes matter with a pressure component along the extra dimension in the brane energy-momentum tensor, the cosmology is of non-standard type. In particular one can get acceleration when a dust of non-relativistic matter particles is the only source for the (modified) Friedman equation. Our equations would seem to violate the conservation of energy-momentum from a 4D perspective, but in 5D the energy-momentum is conserved. One could write down an effective conserved 4D energy-momentum tensor attaching a "dark energy" component to the energy-momentum tensor of matter that has pressure along the extra dimension. This extra component could, on a cosmological scale, be interpreted as matter-coupled quintessence. We comment on the effective 4D description of this effect in terms of the time evolution of a scalar field (the 5D radion) coupled to this kind of matter.

The quest for a modified theory of gravity has taken renewed impetus with the recent cosmological observations implying that the expansion of our universe is accelerating [1]. And given the technical and conceptual difficulties that modifications of 4D General Relativity present the braneworld idea is also a natural arena in which to look for non-standard behavior of gravitational dynamics. The best studied case is the codimension one braneworld, where one can find 4D gravity on the brane even in the presence of an infinite extra dimension, when the curvature of the bulk spacetime is negative and its volume finite [2]. Usually the brane is modeled as a distributional source in the energy-momentum tensor (EMT) of zero thickness, and in this case the cosmology has been obtained and analyzed in detail [3, 4]. Unfortunately the non-standard features of the braneworld cosmology (the presence of a "dark radiation" term, to be identified with the mass of a bulk black hole [4]) did not shed any light on the apparently bizarre energy composition of our universe. In this letter we show that when one drops the infinitesimally thin idealization in the modeling of the brane, and one includes the possibility of having a pressure component in the brane EMT along the extra dimension one gets non-standard gravitational dynamics.

Since we are interested in the cosmological behavior of the braneworld we take the metric ansatz

$$ds^2 = n^2(r,t)dt^2 - a^2(r,t)d\mathbf{x}^2 - e^{2\phi(r,t)}dr^2, \tag{1}$$

where the (only) brane will be now an extended object filling the region with $|r| < \epsilon$ and we have taken flat spatial sections for simplicity. Furthermore we assume a Z_2 symmetry with respect to $r = 0$. Now we can obtain the matching conditions, that make reference to the metric and its first derivatives evaluated at $r = \epsilon$, by integrating the $\mu\nu$ Einstein equations (in the form $M_*^3 R^\mu_\nu = T^\mu_\nu - \delta^\mu_\nu T/3$) in the $|r| < \epsilon$ region giving

$$e^{-\phi}\frac{2n'}{n}|_\epsilon = \frac{1}{M_*^3}\left[\frac{2}{3}\rho + p + \frac{1}{3}p_r\right], \tag{2}$$

$$e^{-\phi}\frac{2a'}{a}|_\epsilon = \frac{1}{M_*^3}\left[\frac{1}{3}\left(p_r - \rho\right)\right], \tag{3}$$

where a prime denotes differentiation with respect to r, M_* is the five-dimensional reduced Planck mass and the energy densities ρ, p and p_r are defined as the integration of the

corresponding elements of the full 5D EMT from $r = -\epsilon$ to $r = \epsilon$:

$$\rho \equiv \frac{1}{na^3|_\epsilon} \int_{-\epsilon}^{\epsilon} T_0^0 n a^3 e^\phi dr, \tag{4}$$

$$p \equiv -\frac{1}{na^3|_\epsilon} \int_{-\epsilon}^{\epsilon} T_x^x n a^3 e^\phi dr, \tag{5}$$

$$p_r \equiv -\frac{1}{na^3|_\epsilon} \int_{-\epsilon}^{\epsilon} T_r^r n a^3 e^\phi dr. \tag{6}$$

Once one considers a thick brane these matching conditions are only approximate, as we have neglected terms in the integration of the Einstein equations that are proportional to the derivatives of the metric along the parallel brane coordinates. It can be seen that, provided the brane is *thin* – in a sense that we will define more precisely below, see eq.(18)– these terms are indeed negligible for the cosmological solutions we will consider (in fact they are negligible in most situations, see [5] for a discussion of matching conditions for thick codimension 1 and 2 branes). Notice that we have considered a non-zero T_r^r inside the brane, a component of the brane EMT that is usually taken to be zero in the thin brane limit. However, for a brane with finite width this quantity cannot be assumed to be zero or even time-independent in general, as can be seen from the equation of conservation of the EMT whose r component reads

$$(T_r^r)' + \left(\frac{n'}{n} + \frac{3a'}{a}\right) T_r^r - \frac{n'}{n} T_0^0 - \frac{3a'}{a} T_x^x = 0. \tag{7}$$

We see that when n' and a' are not zero *inside* the brane we also need in general a nontrivial r-dependence of T_r^r.

We will also assume that in the bulk ($|r| > \epsilon$) the EMT is just that of a cosmological constant, $T_N^M = \delta_N^M \Lambda$. With these assumptions we are ready to obtain the equations governing the cosmological evolution of our braneworld. As in the δ-like case we just have to evaluate the rr and $0r$ components of the full 5D Einstein equations at $r = \epsilon$. These are, respectively

$$-3\mathrm{e}^{-2\phi} \frac{a'}{a} \left(\frac{a'}{a} + n'\right) + 3\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = \frac{\Lambda}{M_*^3}, \tag{8}$$

$$\partial_0 \left[-6e^{-\phi} \frac{a'}{a}\right] + 6\frac{\dot{a}}{a} e^{-\phi} \left[n' - \frac{a'}{a}\right] = 0, \tag{9}$$

where a dot denotes differentiation with respect to t, we have taken $n(\epsilon, t) = 1$ and all the functions in here and in the following are evaluated at $r = \epsilon$ unless stated otherwise. We

consider now a splitting of the brane EMT in a constant, time independent background part (the so-called brane tension) and a time-dependent matter contribution (ρ_m) with an arbitrary (but assumed constant) equation of state, so taking

$$\rho = T + \rho_m, \quad p = -T + \omega\rho_m, \quad p_r = -T_r + \omega_r\rho_m, \tag{10}$$

and using our matching conditions, eqs.(2,3), we can expand our equations at first order in ρ_m to get

$$3\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = \frac{1}{12M_*^6}\Big[2(T+T_r)^2 + (T+T_r)(1-3w-4w_r)\rho_m\Big] + \frac{\Lambda}{M_*^3}, \tag{11}$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\left(1+\omega\right)\rho_m - \omega_r\dot{\rho}_m = 0. \tag{12}$$

It is immediate to see that if $\omega_r = 0$ we recover a conventional cosmology (with a cosmological constant given in Planck mass units by $\Lambda_{eff} = \frac{1}{6M_*^6}\left(T+T_r\right)^2 + \frac{\Lambda}{M_*^3}$) up to $\mathcal{O}(\rho_m^2)$ terms as for the infinitesimally thin brane. The only difference with the well studied δ-like brane would be an extra contribution to the inverse Planck mass and cosmological constant that can be taken into account by shifting $T \to T + T_r$. But for a generic value of ω_r these equations look quite different from the conventional cosmological equations obtained in 4D General Relativity. Notice that in particular the second of these equations would seem to violate the 4D EMT conservation for a matter component with nonzero ω_r. One can integrate this equation to

$$\rho_m = \rho_{m0}a^{-3\frac{1+\omega}{1-\omega_r}}, \tag{13}$$

where ρ_{m0} is a constant. Using this in (11) and taking $\Lambda_{eff} = 0$ we can get our modified Friedman equation as

$$H^2 = \mathcal{C}a^{-4} + \frac{1}{3M_p^2}\left(1-\omega_r\right)\rho_{m0}a^{-3\frac{1+\omega}{1-\omega_r}}, \tag{14}$$

where $H = \frac{\dot{a}}{a}$, $\mathcal{C}$ is a constant of integration and $M_p^2 = \frac{6M_*^6}{T+T_r}$. We see that an energy component with an equation of state that has ω_r different from zero acts as a fluid with an effective equation of state given by

$$\omega_{eff} = \frac{\omega+\omega_r}{1-\omega_r}. \tag{15}$$

In particular we can get an accelerating universe ($\omega_{eff} < -1/3$) in the case of having just a matter component as the only source of the Friedman equation having $\omega = 0$ if $\omega_r < -1/2$.

Formally, the extra-dimensional pressure, p_r, enters all the equations as an extra contribution to the brane EMT with a cosmological constant-like equation of state. So we would obtain the same cosmological equations for an infinitesimally thin brane if we considered an extra contribution to its energy-momentum tensor like

$$T_M^{N(br)} = diag(\rho, -p, -p, -p)\delta(r) - \delta_M^N p_r \delta(r). \tag{16}$$

So taking this redefined brane EMT one gets conventional conservation of energy and momentum. In this case we could see our system (when $p_r = -T_r + \omega_r \rho_m$) as a two fluid one where, besides an extra contribution to the cosmological constant given by T_r, there is an energy interchange between conventional matter and a quintessence-like fluid that has pressure an energy density proportional to the matter energy density, like in the asymptotic solutions of models of interacting quintessence [6, 7].

For the thick brane one can check that the time component of the full 5D EMT conservation equation is indeed compatible with a time dependent p_r but only when the brane is thick and ϕ is also time dependent, since its integration inside the brane reads

$$\begin{aligned} &\frac{1}{na^3|_\epsilon} \int_{-\epsilon}^{\epsilon} \left[\dot{T}_0^0 + 3\frac{\dot{a}}{a}\left(T_0^0 - T_x^x\right) + \dot{\phi}\left(T_0^0 - T_r^r\right) \right] na^3 e^\phi dr \\ &\simeq \dot{\rho}_m + 3H(1+w)\rho_m - \frac{1}{na^3|_\epsilon} \int_{-\epsilon}^{\epsilon} T_r^r \dot{\phi} na^3 e^\phi dr = 0, \end{aligned} \tag{17}$$

where in the second line we have approximated $a(r,t) \simeq a(\epsilon, t)$ for $|r| < \epsilon$. Comparing this equation with eq.(12) it is obvious that we need a time dependent ϕ inside the brane (or, equivalently, a variable brane thickness) whenever we have matter with nonzero ω_r. One can also understand a nonzero ω_r as an extra coupling of matter to the radion field, since after all p_r is nothing but the result of taking a variation of the brane Lagrangian with respect to $e^{2\phi}$, the rr component of the metric.

The obvious question is now: is it possible to have matter on the brane such that its equation of state has w_r different from zero? We can argue that this is the case when these particles are KK "modes" of matter localized on the brane. We use the quotes because by KK modes we mean now modes such that part of its mass comes from having a nontrivial profile *inside* the brane. It is easy to check that a KK mode of a scalar field in case of having a flat circular extra dimension has $\omega_r = m_{KK}^2 / \left(m_{KK}^2 + m^2\right)$ where m_{KK} is a mass gained from a KK mechanism and m is a "bare" 5D mass. However we want to focus now on modes

that are confined on the brane but gain a mass from having a non-trivial profile in the extra-dimension. The existence of such modes can be expected generically in topological defects like a domain wall [8, 9] or a vortex [10], and they cannot be obtained in the thin limit since, as is no surprise, they are non-analytical in the defect width. The modes studied in [8, 10] are excitations of the fields that create these topological defects and can also be understood as oscillations of the width of the defects. So whenever there is a field theory description of our brane one would expect that this kind of excitations, that imply oscillations of the brane width, have a non-negligible (with respect to the energy density) pressure along the extra dimension.

So we have seen that we can understand the cosmology of a brane universe filled with a dust of particles that have pressure along the extra dimension as a quintessence-like one. Now the relation of this “effective quintessence fluid” energy density and pressure to the matter one can be easily explained, suggesting a solution to the coincidence problem. But how could we understand this behavior from the dynamics and interactions of the particles that form this dust? As we have said, comparing eqs.(12) and (17) we see that we can atribute the apparent violation of the EMT conservation equation for matter with non-zero ω_r to a time dependent brane thickness, or radion field. Also using these equations we can estimate the radion time dependence as $\dot{\phi} \sim \omega_r \dot{\rho}_m / T_r \sim \omega_r H^3 M_p^2 / T_r$. It is worth mentioning that one can expect a conformal coupling for the radion in a 4D effective theory (see for instance [11]), and a time dependent conformally coupled scalar field would generate a time dependent mass and an effective (anti)friction force for these particles in the 4D theory (see also [12]). This would in turn produce an apparent non-conservation (through an effective pressure) in the 4D EMT [13], but the time dependence of the radion field seems to be too small to attribute the non-standard features of the cosmology to this effect. So a potential for the radion might be relevant in an effective 4D description of this cosmology in terms of interactions of these particles with a scalar field, and this potential should have its origin in the physics generating the domain wall.

The estimation of the radion time dependence allows us to be a bit more specific on size of the terms that we neglected in the matching. Such terms are proportional to the integral inside the brane of the 4D curvature or $\ddot{\phi}$, that are of order $\sim H^2/M_b$ and $\sim \omega_r H^4 M_p^2/(T_r M_b)$, respectively, and we have defined $M_b \equiv 1/\epsilon$. Using now eqs.(13,14) and comparing these terms with the ones we kept in the matching we see that the term coming from the integration

of $\ddot{\phi}$ is negligible (unless T_r is extremelly small), while the term coming from the integration of the 4D curvature is negligible as long as

$$\frac{T + T_r}{M_b} \ll M_*^3, \tag{18}$$

that we regard as our definition of a thin brane.

Now assuming that the Dark Matter particles have a large negative value of ω_r one might build up a model to address the coincidence problem. In this kind of model one would get acceleration when the Dark Matter particles started dominating the energy density of the universe. One should perform numerical simulations in the effective 4D theory describing this braneworld to see how such a model fits the cosmological observations, but recent studies show that models of interacting quintessence can fit very well CMB [14] and structure formation [15] data (see also [16]). The most recent supernova data might however get the simplest scenario into trouble [17], due to the need of a decelerated matter dominated phase at high redshifts.

To sum up: we have obtained the matching conditions for a thick codimension one brane. Allowing for matter sources that have a nonzero pressure component along the extra dimension we have seen that our cosmological equations get modified in a way that allows acceleration even if $\omega = 0$. We have argued that particles having non-negligible pressure along the extra dimension can be associated with excitations of the domain wall impliying a variable brane thickness. In fact eq.(17) has the clear interpretation of relating the apparent non-conservation of the energy momentum-tensor for these particles to a time-dependent radion field inside the brane, or brane thickness. Our results open the possibility of obtaining from the braneworld a natural solution to the coincidence problem, very much along the lines of the quintessence models of [6, 7] (in its asymptotic limit). But in our case the time dependent scalar field is identified with an extra dimensional component of the metric, instead of being the string theory dilaton used in [7]. It would be nice to obtain the 4D effective theory describing these particles from a direct dimensional reduction of the braneworld taking into account the radion field. Keeping track of the higher dimensional diffeomorphisms that can be associated with the conformal invariance of the 4D effective action like in [11] one might end up with a quintessence model along the lines of [6, 7] or with a description in terms of effective forces of the type discussed in [13]. Another necessary development would be to find a higher dimensional Lagrangian with domain wall

solutions bounding stable particles with sizable and negative ω_r, realizing this mechanism. Such a model could provide the first natural microphysical realization of a quintessence-like cosmology that addresses the coincidence problem.

Acknowledgments

We would like to thank PPARC and DOE for financial support. I.N. would like to thank Fermilab for hospitality during the completion of this work.

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